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Fuzzy Hybrid Control of Flexible Inverted Pendulum (FIP) System using Soft-computing Techniques

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ABSTRACT

The cart-and-pendulum system is a highly nonlinear and under-actuated system that is a great source of interest and motivation for researchers all over the world. There are various configurations of the cart-and-pendulum system that finds wide applications in areas of manufacturing, robotics and control. This paper presents an offline mode control of the Flexible Inverted Pendulum (FIP), which is an extended version of conventional rigid-link pendulum system. The flexibility induced in the pole gives an additional degree of freedom to the system. The nonlinear differential equations were derived using Newton's second law of motion. The study inculcates Fuzzy-based Adaptive Neuro Fuzzy Inference System (ANFIS) controllers for achieving the desired objective. The performance of controllers was measured and compared in a Matlab-Simulink environment. The study considered the effect of friction during motion of the proposed system. The results clearly showed that the ANFIS controller effectively mimics and optimises the behaviour of the Fuzzy controller. The number of Fuzzy rules were also significantly reduced using the ANFIS techniques.

Keywords: FIP, Fuzzy logic, ANFIS, membership function, Matlab-Simulink

INTRODUCTION

The cart-and-pendulum system belongs to a category of highly nonlinear, multivariable and intricate systems that find extensive applications in industry (Prasad et al., 2014). The cart-and-pendulum system comprises a rigid pole hinged to a movable cart that exists in various configurations (Soto & Campa, 2015). It is a highly dynamic system that mimics the behaviour of many practical systems like elastic columns (Gao, 2012; Azimi & Koofigar, 2015), rockets, walking robots etc. (Loram & Lakie, 2002). In this paper, we

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have considered an offline mode control of the Flexible Inverted Pendulum (FIP) through Fuzzy-based ANFIS controllers. The elasticity induced in the pendulum makes its dynamics more complex compared to the conventional rigid pendulum system (Xu & Yu, 2004). It is an important factor, which is to be considered during designing of flexible structures and buildings (Kawaji & Kanazawa, 1991). The literature suggests that an ample amount of work has been carried out for controlling these nonlinear systems. According to Itik and Salamci (2006) a Sliding Mode Control (SMC) can be successfully applied to damp vibrations of an elastic beam acting as a cantilever. A mathematical model of the beam was developed using ordinary differential equations. The experiments performed on the beam system illustrated the satisfactory performance of the SMC controller.

Kong (2009) proposed an intelligent Fuzzy proportional-derivative control of the Flexible Inverted Pendulum (FIP) system. The author used the Euler-Lagrange energy technique for modelling of the proposed system. The study further compared the Fuzzy proportional-derivative technique with the classical proportional-derivative approach. Tang and Ren (2009) presented a dynamic model of the planar Flexible Inverted Pendulum using the Floating Frame Of Reference Formulation (FFRF) technique. The state space equations for the proposed system were derived and validated by means of a simple low-pass filter. Zarafshan and Moosavian (2011) proposed a Rigid-Flexible Interactive Dynamics Modelling (RFIM) for control of the multi-body systems. The proposed approach combines the Lagrange and Newton-Euler methods for developing motion equations of rigid and flexible members. The results revealed the accuracy of the proposed approach for dynamic modelling of mobile robotic systems. Bui et al. (2011) designed three controllers, namely the OFCHA (Optimal Fuzzy Control Using Hedge Algebras), FCHA (Fuzzy Control Using Hedge Algebras) and CFC (Conventional Fuzzy Control) for control of nonlinear systems. The proposed controllers were applied to control the damped elastic-jointed inverted pendulum under periodic follower force at the upright position. The results that showed better performance of the OFCHA and FCHA controllers compared to the CFC controllers.

Litak and Coccolo (2012) presented the dynamics of the elastic inverted pendulum with tip mass under horizontal harmonic excitation. The authors examined the Melnikov Chaos and Stationary Chaos for fractal borders between basins of attraction. Yu et al. (2012) considered the mathematical model of the Linear Quadratic Regulator (LQR)-based Sugeno Neural Controller to control the flexible double-inverted pendulum. The simulation results showed better performance of the neural controller compared to the LQR controller. A Fuzzy Takagi-Sugeno-Kang (TSK) controller to stabilise flexible rotary joint manipulator was proposed by Akbari et al. (2012). A solenoid spring was connected between the actuator output and joint input, thus inducing flexibility in the system. Experimental results showed excellent performance of the proposed controller in controlling the flexible joint manipulator. Abdullahi et al. (2013) presented the fuzzy control and pole placement control of vibration and tip deflection of a single link flexible manipulator. The fuzzy controller provides damping to the joint, which minimises vibrations and tip deflection, whereas pole placement control keeps the system pole at a desired location.

Semenov et al. (2015) examined control of the elastic inverted pendulum subjected to hysteretic nonlinearity at the point of suspension. The study considered an algorithm based on a bionic model for control of the proposed system. Numerical simulations were further performed to verify the results. The elastic inverted pendulum as a nonlinear energy harvester model was proposed by Halvorsen and Litak (2015). The authors derived a set of Fokker-Planck equations to obtain an expression for probability density of the system. A dynamic equilibrium criteria for control of the elastic inverted pendulum with tip mass was examined by Gorade et al. (2015). The mathematical model of the proposed system was framed using the Euler-Lagrange analysis. The results collected after simulation of the above system were further compared with the experimental data of the actual plant.

In a study by Shahbazi et al. (2016), the control dynamics of the Spring-Loaded Inverted Pendulum (SLIP) at steady and transition states were examined. The approach realised the behaviour of the proposed system during running, walking and walk-run transitions. The study further utilised different gaits generated by means of hybrid automation to illustrate synthesis of behaviour for the SLIP system.

MODELLING OF FIP SYSTEM

The mathematical model of the FIP system was built combining the dynamic behaviour of both the rigid and beam theories (Dadios, 1997). The nonlinear differential equation for the FIP was derived using Newton's second law of motion. The deflection of the elastic pole gives an additional degree of freedom to the system. A free-body-diagram of the FIP system illustrating complete dynamics is shown in Figure 1 (Bayramoglu & Komurcugil, 2013). The FIP system comprises a flexible pendulum of mass (m) and length (L) hinged to a movable cart of mass (M). A control force, F, is required to drag the cart in a horizontal direction. The angles of the rigid and flexible pendulum from the vertical axis were θ and θ_t respectively. The other important attributes considered were breadth of pendulum (b), depth of pendulum (d), elasticity of pendulum (e), friction coefficient (u) and acceleration due to gravity (g). The values of the different attributes considered for simulation are presented in Table 1.

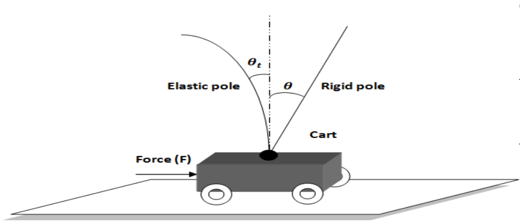

Figure 1. FIP on cart

Table 1
Values of Different Attributes for Simulation

S. No	Attribute	Value
1	Mass of pendulum (m)	0.8 kg
2	Mass of cart (M)	3.0 kg
3	Length of pendulum (L)	1.5 m
4	Breadth of pendulum (b)	0.05 m
5	Depth of pendulum (d)	0.008 m
6	Elasticity of pendulum (e)	0.18 Pascal
7	Gravity (g)	9.81 m/s^2
8	Friction coefficient (u)	0.1 Nm/s

The equations for motion of the FIP system were derived as follows:

i. Equation for rigid pendulum

$$\ddot{\theta} = \frac{(m+M)g\sin\theta - \cos\theta\{F - u.m.g + \theta^2.m.r\ (\sin\theta + u\cos\theta)\}}{\frac{4}{3}(m+M)r - mr\cos^2\theta + u.m.r.\cos\theta.\sin\theta} \tag{1}$$

ii. Equation for flexible pendulum

$$\ddot{\theta}_t = \frac{\ddot{\theta}(1+K\cos\theta)}{1+(K\sin\theta)^2} + \frac{-K\,\dot{\theta}^2\sin\theta\{1+K^2(1+\cos^2\theta)\}}{\{1+(K\sin\theta)^2\}^2} \tag{2}$$

iii. Equation for cart

$$\ddot{x} = \frac{\{F - u.m.g - \ddot{\theta}.m.r\ (\cos\theta - u\sin\theta) - \dot{\theta}^2.m.r\ (-\sin\theta - u\cos\theta)\}}{(m+M)} \tag{3}$$

where K $\frac{12L^2mg}{8ebd^3} =$ and r = $\frac{L}{2}$

The above equations were used for building a Simulink model of the FIP system as given in Figure 2.

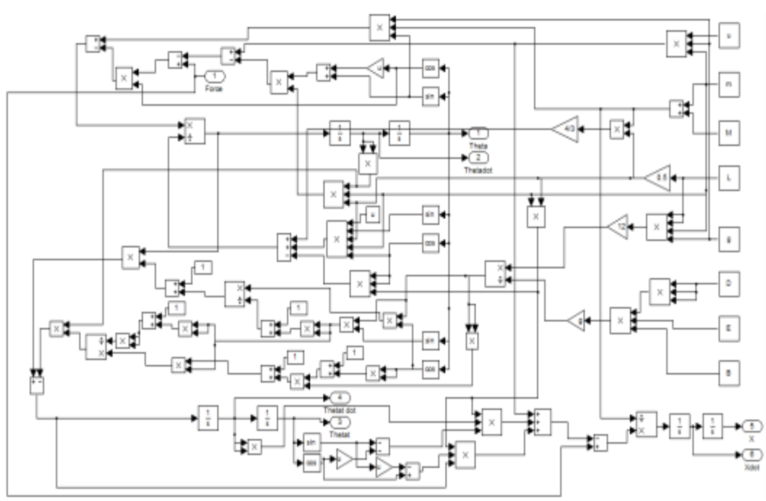

Figure 2. Simulink of FIP system

Fuzzy-Based ANFIS Control of the FIP System

The Fuzzy logic theory is a reasoning-based soft-computing technique widely used by researchers in control of nonlinear processes (Wang & Tan, 1997). It was initially introduced by Zadeh (1965), who highlighted the basic concept of Fuzzy representation and the Fuzzy Coordinate system. Fuzzy logic is an important artificial tool comprising of IF-THEN based Fuzzy rules designed based on expert knowledge (Alcala et al., 2009). A basic Fuzzy architecture comprises a fuzzification interface that receives crisp values as input and converts it into Fuzzy input. The Fuzzy input is subjected to the Fuzzy inference engine, which converts it to Fuzzy output by a set of Fuzzy rules. The Fuzzy output thus obtained is further converted into a crisp value using a defuzzification interface (Bordon et al., 2000). A schematic view of the Fuzzy architecture is shown in Figure 3.

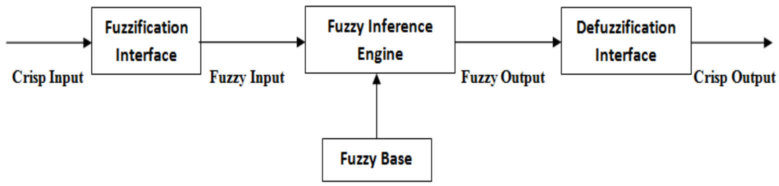

Figure 3. Fuzzy architecture

The fuzzification of input variables was achieved using nine Gaussian-shaped membership functions as shown in Figure 4. The linguistic variables considered for defining of membership functions are as follows: Negative Low-NL, Negative Medium-NM, Negative Small-NS, Zero-ZE, Positive Small-PS, Positive Medium-PM and Positive Large-PL. These linguistic variables were used for building of IF-THEN Fuzzy rules for the controller as given in Table 2.

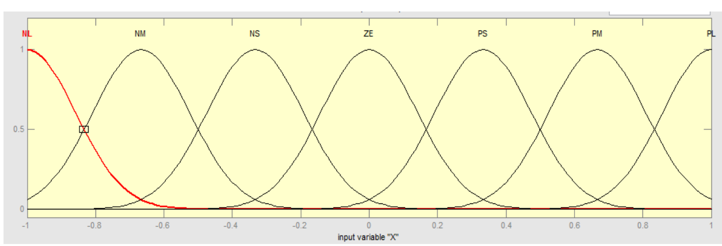

Figure 4. Membership functions designed for cart controller

Table 2
Fuzzy Control Rules for Cart and Pendulum Controllers

Force (F)		Cart velocity						
Cart position		NL	NM	NS	ZE	PS	PM	PL
	NL	NL	NL	NM	NS	ZE	ZE	ZE
	NM	NL	NM	NS	NS	ZE	ZE	ZE
	NS	NM	NM	NS	ZE	ZE	PS	PM
	ZE	NM	NS	ZE	ZE	PS	PS	PM
	PS	NM	NS	ZE	PS	PS	PM	PL
	PM	NS	ZE	PS	PS	PM	PM	PL
	PL	NS	ZE	PS	PM	PM	PL	PL

Table 2 *(continue)*

Force (F)		Pendulum angular velocity						
Pendulum angle		NL	NM	NS	ZE	PS	PM	PL
	NL	NL	NL	NM	NS	ZE	ZE	ZE
	NM	NL	NM	NS	NS	ZE	ZE	ZE
	NS	NM	NM	NS	ZE	ZE	PS	PM
	ZE	NM	NS	ZE	ZE	PS	PS	PM
	PS	NM	NS	ZE	PS	PS	PM	PL
	PM	NS	ZE	PS	PS	PM	PM	PL
	PL	NS	ZE	PS	PM	PM	PL	PL

The fuzzy control rules mentioned above for the cart-and-pendulum sub-system were designed by experts based on their experience and prior knowledge of the proposed system. A set of 49 IF-THEN Fuzzy rules were developed to effectively control the proposed system.

The Fuzzy rules mentioned in Table 2 were further represented in a three-dimensional representation by a Surface viewer as shown in Figure 5.

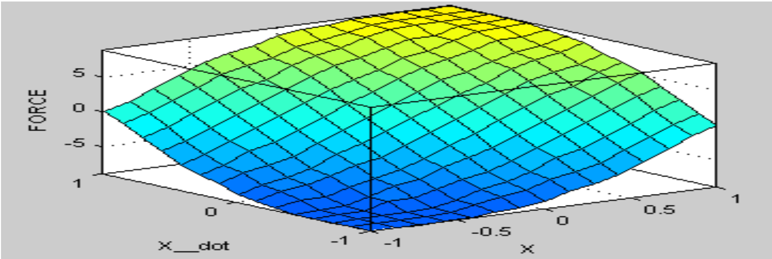

Figure 5. Surface viewer for cart controller

Artificial neural networks comprise numerous inter-connected information processing elements called neurons. These are arranged in a pattern similar to the cerebral cortex portion of the human brain. These networks were organised in different layers as shown in Figure 6. The layers were connected to each other with the help of nodes having an activation function. Inputs were given to the network via an input layer that was further linked to hidden layers. Hidden layers performed processing on the inputs by adjusting their connection weights. The outputs were generated from the network with the help of output layers.

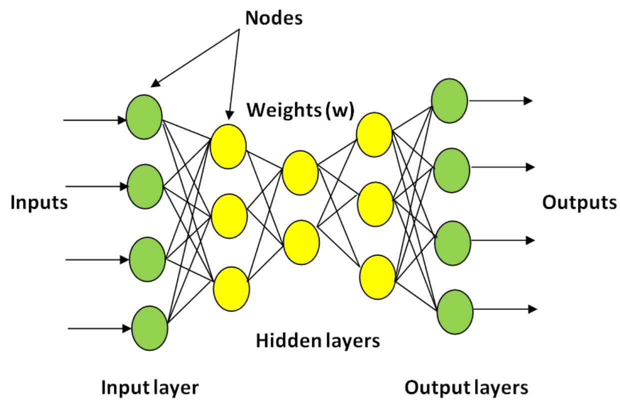

Figure 6. Neural network architecture

ANFIS belongs to a class of adaptive networks that are functionally equivalent to Fuzzy inference systems (Buragohain, 2008). These are hybrid learning algorithms widely used to optimise and mimic responses of nonlinear controllers (Tatikonda et al. 2000). The training in ANFIS was performed using a hybrid learning algorithm (Shoorehdeli et al. 2009) that uses the least square method (Kubacek et al., 1978) and back-propagation learning algorithm (Li et al. 2012) for its tuning. In this study the results from simulation of a Fuzzy controller were collected and applied for training of the ANFIS controller. A total of 176 data samples were collected and stored in M-file for training. The number of training epochs and error tolerance was set to 50 and 0, respectively. A view of loading and training of data samples in ANFIS is shown in Figure 7. and Figure 8,

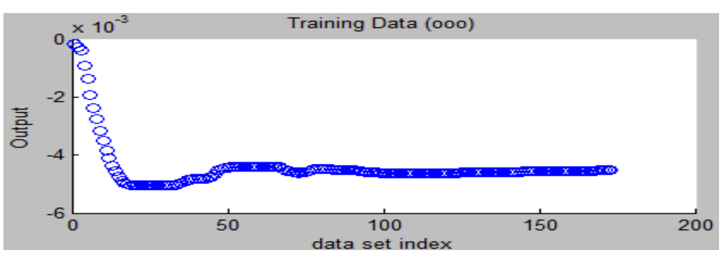

Figure 7. Loading of data sets for cart controller

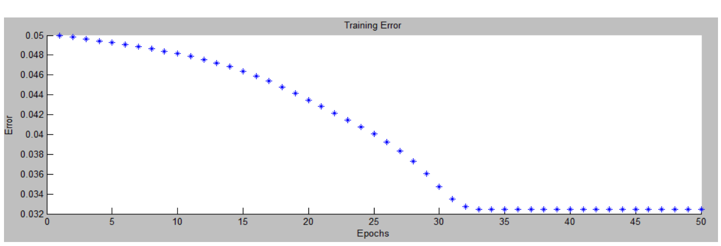

Figure 8. Training of data sets for cart controller

During training, grid partition method generates an initial Fuzzy Inference Structure (FIS) as shown in Figure 9. The errors obtained after training of five Gaussian shaped membership functions for the cart-and-pendulum controller were 3.381e-005 and 0.000224, respectively. A view of the modified membership function and surface viewer after training for the cart controller can be seen in Figure 10 and Figure 11, respectively.

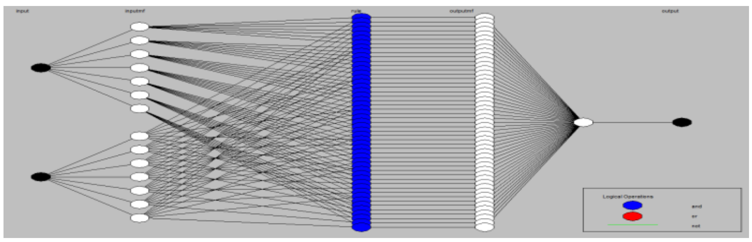

Figure 9. ANFIS architecture generated after training

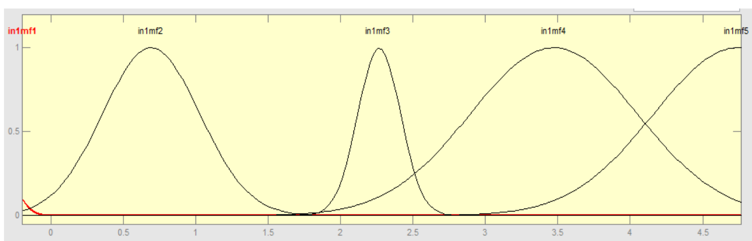

Figure 10. Modified membership functions after training

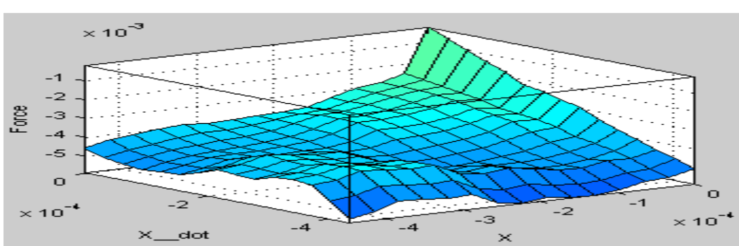

Figure 11. Modified surface viewer after training

SIMULATION RESULTS

The simulations were performed in Matlab, with a simulation time of 10 seconds. A graphical view of the simulation responses and their comparison are given below.

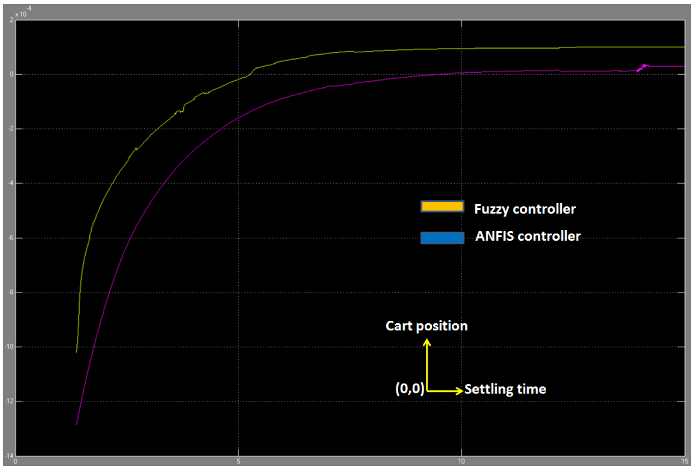

Figure 12. Simulation response for cart position (*x*)

Table 3
Results Comparison for Cart Position

Controller	Settling time (sec)	Max. overshoot (degree)	Steady State Error
ANFIS	7.0 sec	-12.8° to 0.2×10^{-4}	0
Fuzzy	7.0 sec	-10.2° to 1×10^{-4}	0

The simulation results given in Table 3 clearly showed that the ANFIS controller effectively mimiced the behaviour of a Fuzzy controller. Both the controllers took almost the same amount of time to stabilise a response for the cart position. The ANFIS controller showed a better maximum overshoot response compared to other controller. It was also observed from the results given above that both the controllers had an excellent steady state response.

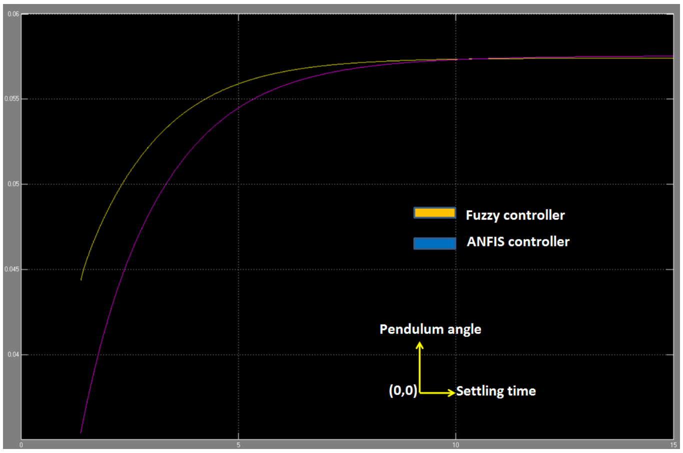

Figure 13. Simulation response for pendulum angle (θ)

Table 4
Results Comparison for Cart Velocity

Controller	Settling time (sec)	Max. overshoot (degree)	Steady State Error
ANFIS	7.5 sec	0.00575°	0
Fuzzy	7.5 sec	0.0058°	0

The simulation results given in Table 4 showed that both the controllers were able to stabilise the complete system in 7.4 seconds. The ANFIS controller resulted in comparatively better overshoot compared to the Fuzzy controller. Excellent steady state response was obtained for both the controllers.

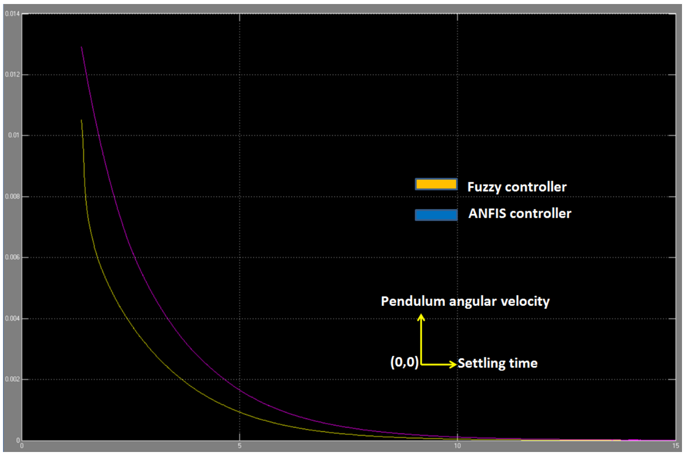

Figure 14. Simulation response for pendulum angular velocity ($\dot{\theta}$)

Table 5
Results Comparison for Pendulum Angle

Controller	Settling time (sec)	Max. overshoot (degree)	Steady State Error
ANFIS	8.0 sec	0.013°	0
Fuzzy	8.5 sec	0.0105°	0

The simulation results given in Table 5 indicated that the settling time was reduced by 0.5 seconds using the ANFIS controller. It was also observed that better maximum overshoot response was obtained using the Fuzzy controller. Again, both the controllers showed excellent response of steady state error.

CONCLUSION

This paper highlighted soft-computing based control of the Flexible Inverted Pendulum (FIP) system, which is an upgraded version of the conventional rigid-link pendulum system. The study explained in brief the methodology and procedure opted for designing a Fuzzy and

Fuzzy-based ANFIS controller. The results derived from using the Fuzzy controller were collected and applied for tuning of the ANFIS controller. The ANFIS showed excellent training capacity and gave a minimal training error of 3.381e-005 and 0.000224 for cart-and-pendulum controllers, respectively. The ANFIS controller not only tuned, but also reduced the number of if-then Fuzzy rules by using only five Gaussian shaped membership functions for its training. It was clearly observed from the results (refer Table 3 to Table 5) that the ANFIS controller effectively mimicked the behaviour of a Fuzzy controller. It was also observed that the maximum overshoot responses using the ANFIS were better except for the case of pendulum angular velocity. Finally, it was observed that both the controllers showed excellent response towards steady state error. As an extension to future works, several other control algorithms like the Proportional-Integral-Derivative (PID), genetic algorithm and particle swarm optimisation are also under consideration for control of the proposed system.

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